

## 4.1 Videos Guide

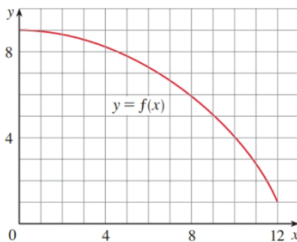
### 4.1a

- Introduction to the concept of areas and distances

### 4.1b

Exercise:

- (a) Use six rectangles to find estimates of each type for the area under the given graph of  $f$  from  $x = 0$  to  $x = 12$ .
  - (i)  $L_6$  (sample points are left endpoints)
  - (ii)  $R_6$  (sample points are right endpoints)
  - (iii)  $M_6$  (sample points are midpoints)
- (b) Is  $L_6$  an underestimate or overestimate of the true area?
- (c) Is  $R_6$  an underestimate or overestimate of the true area?
- (d) Which of the numbers  $L_6$ ,  $R_6$ , or  $M_6$  gives the best estimate? Explain.



### 4.1c

Definition: (area)

- The area  $A$  of the region  $S$  that lies under the graph of the continuous function  $f$  is the limit of the sum of the areas of approximating rectangles:
$$A = \lim_{n \rightarrow \infty} R_n = \lim_{n \rightarrow \infty} [f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x]$$
ALSO 
$$A = \lim_{n \rightarrow \infty} L_n = \lim_{n \rightarrow \infty} [f(x_0)\Delta x + f(x_1)\Delta x + \cdots + f(x_{n-1})\Delta x],$$
where  $R_n$  indicates rectangles whose heights are given using the right endpoints of subintervals and  $L_n$  uses left endpoints of subintervals

Exercises:

- Use the definition of area to express the area under  $f$ .
$$f(x) = x^2 + \sqrt{1 + 2x}, \quad 4 \leq x \leq 7$$

### 4.1d

- Determine a region whose area is equal to the given limit. Do not evaluate the limit.

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{3}{n} \sqrt{1 + \frac{3i}{n}}$$

- The velocity graph of a car accelerating from rest to a speed of 120 km/h over a period of 30 seconds is shown. Estimate the distance traveled during this period.

